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Emergence of a Big Bang singularity in an exact string background

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The origin of Big Bang singularity in $3+1$ dimensions can be understood in an exact string theory background obtained by an analytic continuation of a cigar like geometry with a nontrivial dilaton. In a T-dual conformal field theory picture there exists a closed string tachyon potential which excises the singular space-time of a strongly coupled regime to ensure that a higher dimensional universe has no curvature singularity. However in $3+1$ dimensions the universe exhibits all the pathology of a standard Big Bang cosmology. The emergence of a singularity now owes to a higher dimensional orbifold singularity which does not have a curvature singularity in higher dimensions, suggesting that close to the compactification scale an effective description of $3+1$ dimensions breaks down and bouncing universe emerges in 5 and higher dimensions.

For any equation of state obeying the strong energy condition $p > -\rho/3$, regardless of the geometry (flat, open, closed) of the universe, the scale factor of the universe in a Friedmann Robertson Walker (FRW) metric vanishes at $t = 0$, and the matter density diverges. In fact all the curvature invariants, such as R, $\Box R, ...$, become singular. This is the reason why it is called the *Big Bang singularity problem* [1].

There has been many attempts to resolve this issue by invoking anisotropic stresses, self regenerating universe (during inflation) quantum cosmology, etc. (see [2]) but resolving the space-like singularity is particularly hard, especially in the context of a flat universe *.

The aim of this paper is to show the *emergence* of a Big Bang singularity in $3+1$ dimensions within a string theory setup, where the scale factor of a flat, homogeneous and isotropic, Friedmann Robertson Waker (FRW) metric undergoes a de-accelerating expansion, and the scale factor of the universe vanishes in finite time.

However we shall argue that this is an *effective* description of the universe and as we approach near the compactfication scale (which could be as large as the four dimensional Planck scale), the Big Bang singular region unfolds to a $4+1$ dimensional world with a *bouncing* cosmology. The origin of Big Bang singularity is now clear, it is due to a *boost orbifold singularity* in higher dimensions and not as a *curvature* singularity.

In order to realize a cosmology which is free from curvature singularity in *higher dimensions*, we wish to avoid space-like singularity. We would also wish to have a background geometry which has an exact Conformal Field Theory (CFT) description. In which case the space-time is exact in all orders in α'. In addition if the exact CFT requires a nontrivial dilaton varying in space-time, then its growth needs to be bounded from above for the quantum corrections to be suppressed.

In order to illustrate our setup let us consider a two-dimensional cigar-like geometry with a space varying dilaton [5, 6, 7]

$$\begin{aligned} ds^2 &= k\left[dr^2 + \tanh^2 r d\phi^2\right] \\ \Phi - \Phi_0 &= -\log\cosh r\,, \end{aligned} \tag{1}$$

where ϕ is a periodic coordinate with $\phi \sim \phi + 2\pi$. In string theory the cigar corresponds to an exact conformal field theory given by the coset $(SL(2,\mathbb{C})/SU(2))/U(1)$, where the parameter k in the metric corresponds to the level of the $SL(2)$ current algebra. The central charge is given by $c = 3k/(k-2) - 1$ in the bosonic case and $c = 3(k+2)/k$ in the supersymmetric case [8].

The metric Eq. (1) is exact in the supersymmetric case and receives $\mathcal{O}(1/k)$ corrections in the bosonic case. The dilaton is bounded from above and the string coupling $g_s = e^{\Phi}$ remains small in the entire space by choosing $e^{\Phi_0} \ll 1$.

Furthermore it has been conjectured that the cigar CFT is equivalent to the Sine-Liouville model – FZZ duality [9, 10] (see also [11]). The sine-Liouville model is defined by the Lagrangian

$$\mathcal{L} = \frac{1}{4\pi}\left[(\partial x_1)^2 + (\partial x_2)^2 + Q\hat{R}x_1 + \lambda\, e^{-x_1/Q}\cos Rx_2'\right], \tag{2}$$

where we have defined the T-dual coordinate $x_2' \equiv x_{2L} - x_{2R}$ while $x_2 = x_{2L} + x_{2R}$. This is a linear dilaton CFT with $\Phi - \Phi_0 = -Qx_1$ and the sine-Liouville interaction

$$T(x_1, x_2') = \lambda\, e^{-x_1/Q}\cos Rx_2'\,, \tag{3}$$

depicts a closed string tachyon condensation. The condensate is exponentially localized, i.e. semi-localized in the x_1 direction and has a winding in the x_2 direction.

The parameters of this theory are related to the level k of the cigar CFT by $Q^2 = 1/(k-2)$ and $R = \sqrt{k}$. An analogous conjecture relates $N = 2$ Liouville theory [12] to the supersymmetric version of the cigar coset CFT [13, 14, 15, 16], in which case $Q^2 = 1/k$. In the Sine-Liouville model x_2 is a periodic coordinate with period

* In the context of a closed universe where the curvature term acts as a “source” for negative energy density in the Hubble equation, one can obtain bouncing solutions [3]. A particularly interesting proposal of a non-singular bouncing cosmology in a flat geometry has been made in [4], where non-perturbative correction to an Einstein Hilbert action leads to an asymptotically free gravity and also ghost free.

$2\pi R$. In the asymptotic weakly-coupled region both the cigar and Sine-Liouville model look like a cylinder with a linear dilaton, and the coordinates are identified as $r \sim Qx_1$ and $\phi \sim x_2/\sqrt{k}$ for large k.

The T-dual sine-Liouville description illustrates a string theory mechanism of the singularity resolution, see Fig.1 [17]. Before the tachyon $T(x_1, x_2')$ condenses, i.e. $\lambda = 0$, strings can propagate into $x_1 \to -\infty$, where the string coupling $g_s = e^{\Phi}$ blows up. The space-time has a region of strong coupling singularity. By condensing the tachyon, i.e. $\lambda \neq 0$, the tachyon wall $T(x_1, x_2')$ prevents strings from propagating into the strong coupling region. The singularity is excised that way. Via FZZ duality the tachyon condensation manifests itself as a cigar geometry. In this geometric picture the space-time terminates before the strong coupling singularity develops.

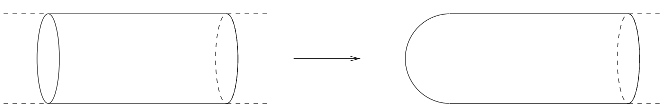

FIG. 1: Semi-localized tachyon condensation turns the cylinder into a cigar.

We wish to apply this mechanism to a cosmological model. Note, however, that the cigar geometry itself is not applicable to a real world cosmology.† In order to obtain a temporal dependence in a time dependent metric, we would require an analytical continuation of the cigar and the sine-Liouville (SL) CFTs [20]:

$$k \to e^{\pi i}k = -k \ , \quad \left\{ \begin{array}{lll} r \to -t \ , & \phi \to -i\varphi & \text{(cigar)} \\ x_1 \to -i\tau \ , & x_2 \to x_2 & \text{(SL)} \ . \end{array} \right. \tag{4}$$

The cigar becomes a time-dependent spacetime

$$\begin{aligned} ds^2 &= k\left[-dt^2 + \tanh^2 t d\varphi^2\right] \\ \Phi - \Phi_0 &= -\log\cosh t \ . \end{aligned} \tag{5}$$

The *sine-Liouville* interaction becomes *sinh-Liouville* and the linear dilaton turns into time-like:

$$\begin{aligned} T(\tau, x_2') &= \lambda\, e^{-\tau/|Q|}\cosh|R|x_2' \\ \Phi - \Phi_0 &= -|Q|\tau \ . \end{aligned} \tag{6}$$

We have now gathered enough ingredients to discuss bouncing cosmology. We focus on the (world-sheet) supersymmetric case. The central charge is $c = 3 - 6/|k|$, or equivalently $\hat{c} = 2 - 4/|k|$. Furthermore we will be interested in the case $|k| \gg 1$ so that (i) the curvature is small and (ii) the central charge $\hat{c} \sim 2$ and the rest of the

† It was shown in [19] that gravitons could be localized in a four dimensional submanifold at the tip of the cigar. It was then suggested that the cigar CFT may be useful for the brane-world cosmology.

space-time can be chosen approximately to be flat, e.g. $\mathbb{R}^3 \times T^5$ with $\hat{c}_{\mathbb{R}^3\times T^5} = 8$.

Our initial background in the *string frame* is $\mathbb{R} \times S^1 \times \mathbb{R}^3 \times T^5$ with a time-like linear dilaton,

$$\begin{aligned} ds^2 &= -d\tau^2 + dx_2^2 + ds^2_{\mathbb{R}^3\times T^5} \\ \Phi - \Phi_0 &= -|Q|\tau \ , \end{aligned} \tag{7}$$

where x_2 is the T-dual of x_2' and $x_2 \sim x_2 + 2\pi\beta$.‡

In the Einstein frame ($G_E = e^{-4(\Phi-\Phi_0)/(D-2)}G_S$, where G_E and G_S correspond to Einstein and string frame metrics, respectively), we have

$$\begin{aligned} ds_E^2 &= e^{|Q|\tau/2}\left(-d\tau^2 + dx_2^2 + ds^2_{\mathbb{R}^3\times T^5}\right) \\ e^{\Phi} &= e^{\Phi_0}e^{-|Q|\tau} \ . \end{aligned} \tag{8}$$

This is a Milne-type universe. In the far past $\tau \to -\infty$ the string coupling blows up and the curvature diverges. Note that the strong coupling singularity translates to a singularity of the space-time in the Einstein frame and the singularity is null-type.

We now let the tachyon condense, $T(\tau, x_2') = \lambda\, e^{-\tau/|Q|}\cosh|R|x_2'$. Then via FZZ duality [9, 10] the tachyon condensation is mapped to a change in the $1+1$ dimensional part of the geometry and the dilaton profile as in Eq. (5) §. The metric in the Einstein frame then yields

$$\begin{aligned} ds_E^2 &= (\cosh t)^{\frac{1}{2}}\left[|k|\left(-dt^2 + \tanh^2 t d\varphi^2\right) + ds^2_{\mathbb{R}^3\times T^5}\right] \\ e^{\Phi} &= e^{\Phi_0}(\cosh t)^{-1} \ . \end{aligned} \tag{9}$$

The resulting space-time is significantly deformed at early times $-t \gg 1$ when the tachyon condensation is mostly localized, but asymptotes to the initial background Eq. (8) at late times. Note that there is no longer a strongly-coupled region nor the curvature singularity as a consequence. The (null) singularity at the beginning of the universe is resolved. To see it, let us introduce the time $\bar{t}$ by $d\bar{t} = \sqrt{|k|}\,(\cosh t)^{\frac{1}{4}}\, dt$. Then the space-time takes the form

$$ds_E^2 = -d\bar{t}^2 + a(\bar{t})^2 ds^2_{\mathbb{R}^3\times T^5} + b(\bar{t})^2 d\varphi^2 \ . \tag{10}$$

The scale factor $a(\bar{t})$ behaves as

$$\dot{a}(\bar{t}) = \frac{1}{4\sqrt{|k|}}\tanh t \ , \qquad \ddot{a}(\bar{t}) = \frac{1}{4|k|}\frac{1}{(\cosh t)^{9/4}} > 0 \ . \tag{11}$$

‡ After the analytic continuation, $x_2' = x_{2L} - x_{2R}$ is no longer periodic due to the hyperbolic dependence of the tachyon condensate $T(\tau, x_2')$. However, its T-dual $x_2 = x_{2L} + x_{2R}$ can be still periodic by the identification $x_{2L} \sim x_{2L} + \pi\beta$ and $x_{2R} \sim x_{2R} + \pi\beta$ and so will be the angular coordinate φ.

§ A similar idea was explored in the study of an exact time-dependent string theory background [18].

The universe is accelerating and the evolution is symmetric under $\bar{t} \leftrightarrow -\bar{t}$. At $\bar{t} = 0$ the scale factor $a(\bar{t}) = 1$ and $\dot{a}(\bar{t}) = 0$. The universe bounces from the contracting to the expanding phase. In the infinite past and future the acceleration stops and the space-time asymptotes to the Milne universe.

To summarize, the tachyon condensation has excised the (null) big-bang singularity and as a consequence rendered the universe of the bouncing type. Here we wish to make a few remarks. The scale factor $b(\bar{t})$ shrinks to a zero size linearly in $\bar{t}$ near the bouncing point $\bar{t} = 0$. Since φ is periodic, this renders the universe singular. The singularity is space-like. However, this is not a curvature singularity but that of a boost orbifold $\mathbb{R}^{1,1}/\mathbb{Z}$ [21, 22, 23]. The space-time must be properly extended to the "Rindler wedges" to ensure the unitary evolution of string states propagating through the singularity [20]. However, since our interest is in the states excited only in $\mathbb{R} \times \mathbb{R}^3$ for the purpose of our bouncing cosmology, one may not concern about the extended Rindler regions.

It is also important to note that there are a few potential sources of instabilities: (i) The most serious of them is a large back-reaction due to an infinite blue-shift near the singularity [23, 24, 25, 26]. The instability was argued to be very severe in the classical gravity approximation [24]. However, this is hardly a definitive consensus and a smooth end remains a possibility: The winding strings become massless at the singularity, however, these states were not taken into account in [24]. Moreover, particles and winding strings are produced near the singularity. So it is important to take these effects into account. They may conceivably work as agents for smoothing out the singularity [23]. There is evidence that the Eikonal resummation may render the singularity much milder [26]. So the back-reaction may not be as large as it was thought in [24]. (ii) Although propagating tachyons are absent in the type II string, certain light modes lead to an imaginary part in the one-loop amplitude due to the asymptotic linear dilaton, signaling a non-perturbative instability [20]. However, since the string coupling is small in our setup, the decay rate $\mathcal{O}(e^{-1/g_s^2})$ in this channel is negligible.

Further note that the analytic continuation renders the level k of the coset CFT negative. This implies that the CFT as a world-sheet theory is not unitary, reflecting the presence of a time-like direction. However, the unitarity of our concern is that of the target space-time theory and it is respected, as mentioned above [20].

Let us now consider how the metric appears upon compactification. The eight dimensional part of the space-time is flat $\mathbb{R}^3 \times T^5$. One scenario in our framework is to consider the universe as $4+1$ dimensions with one extra dimension being small ($\varphi \sim \varphi + 2\pi a$ with $a\sqrt{|k|} \ll 1$).

We are then interested in the five dimensional universe compactified on T^5. Upon the dimensional reduction, the radii/scale factors of the compact space, in effect, provide the additional dilaton coupling to the Einstein-Hilbert action. In our case the easiest is to perform the dimensional reduction in the string frame. Since the compact space is flat, the additional dilaton coupling generated is a *constant*, the constant volume of T^5. For convenience, we choose it to be unity. Then the $(4+1)$-dimensional metric in the Einstein frame can be recast simply from the formula $G_E = e^{-4(\Phi-\Phi_0)/(D-2)} G_S$ with $D = 5$ into

$$ds_E^2 = (\cosh t)^{\frac{4}{3}} \left[|k| \left(-dt^2 + \tanh^2 t d\varphi^2 \right) + d\vec{x}_3^2 \right] . \quad (12)$$

Introducing the time $\bar{t}$ by $d\bar{t} = \sqrt{|k|} (\cosh t)^{\frac{2}{3}} dt$, the space-time yields the FRW universe with one extra compact dimension:

$$ds_E^2 = -d\bar{t}^2 + a(\bar{t})^2 d\vec{x}_3^2 + b(\bar{t})^2 d\varphi^2 . \quad (13)$$

The velocity and the acceleration of the 3-spatial part are respectively given by:

$$\dot{a}(\bar{t}) = \frac{2}{3\sqrt{|k|}} \tanh t , \qquad \ddot{a}(\bar{t}) = \frac{2}{3|k|} \frac{1}{(\cosh t)^{8/3}} > 0 . \quad (14)$$

The evolution of the universe is qualitatively the same as in the higher dimensional case, ensuring a non-singular bounce ¶.

If we further compactify the space-time down to $3+1$ dimensions on S^1 in the φ-direction, the universe becomes

$$ds_E^2 = \frac{\sqrt{|k|}}{2} |\sinh 2t| \left(-|k| dt^2 + d\vec{x}_3^2 \right) . \quad (15)$$

Introducing the time $\bar{t}$ by $d\bar{t} = dt\, |k|^{3/4} |\frac{1}{2} \sinh 2t|^{1/2}$, we have the FRW universe

$$ds_E^2 = -d\bar{t}^2 + a(\bar{t})^2 d\vec{x}_3^2 . \quad (16)$$

where the velocity and the acceleration of the scale factor is given by:

$$\begin{aligned} \dot{a}(\bar{t}) &= \frac{1}{\sqrt{|k|}} \coth 2t , \\ \ddot{a}(\bar{t}) &= -\frac{2\sqrt{2}}{|k|^{5/4}} |\sinh 2t|^{-5/2} < 0 . \end{aligned} \quad (17)$$

In this $3+1$ dimensional effective description, our universe mimics a type of hot Big Bang cosmology. There is a Big Bang singularity at $\bar{t} = 0$, the curvature invariants blow up near $\bar{t} = 0$ and the decelerating expansion of the universe is: $a(\bar{t}) \sim \bar{t}^{1/3}$.

However now the origin of Big Bang singularity can be understood very well. This is due to an orbifold singularity in higher dimensions. To illustrate this let us consider

¶ Provided that the aforementioned back-reaction is considerably softened, as discussed above. Note also that the bounce of [22] happens in the fifth S^1-direction. In contrast, the bounce in our model is with respect to the scale factor $a(\bar{t})$ for the flat three dimensional space.

a toy example: If we consider a 5 dimensional flat space, $ds^2 = dr^2 + r^2 d\phi^2 + d\vec{x}_3^2$, and compactify it along the ϕ circle, we find a similar curvature singularity at $r = 0$, reflecting the corresponding coordinate singularity in five dimensions.

So we interpret the Big Bang singularity as a signature of a breakdown of $3+1$ dimensional effective description near $\bar{t} = 0$. In other words, the universe cannot be viewed as $3+1$ dimensional, as one approaches $\bar{t} = 0$. It is only appropriate to consider the universe as $4+1$ dimensional near $\bar{t} = 0$, where the universe is *free* from curvature singularity.

At this point one might worry about the role of a tachyon in $3+1$ dimensions, would it have any cosmological implications. One should note here that $3+1$ dimensional universe and a tachyon description is dual to each other, i.e., there is no dynamical role of a tachyon.

Typical of a Big Bang cosmology in $3+1$ dimensions, all the relevant problems remain such as flatness, homogeneity and isotropy of the universe. Furthermore we have to explain the temperature anisotropy of the cosmic microwave background (CMB) radiation [27]. In order to address these issues the universe must undergo a phase of cosmic inflation (for a review, see [28]).

The cosmic inflation could be triggered within multiple vacua of a string landscape (for a review see [29]) [30] and end in an observable sector via Minimally Supersymmetric Standard Model (MSSM) inflation [31], which ensures correct phenomenology such as observed neutrino masses, baryon number density, and cold dark matter [32]. Otherwise one could as well resort to a curvaton mechanism to explain the observed temperature anisotropy [33].

To summarize, we have constructed a simple toy model of a hot Big Bang cosmology which is embedded in string theory background and has an exact CFT description. In this model the Big Bang singularity is an artifact of higher dimensions, which signals a breakdown of a $3+1$ dimensional description of our universe. Near $\bar{t} = 0$, the correct description of our universe is given by a $4+1$ dimensional bouncing cosmology with a boost orbifold. The Big Bang singularity is now manifested as an orbifold singularity and not as a curvature singularity. In order to match the success of a Big Bang cosmology one would have to introduce a matter sector in the geometry, which would then ensure successful inflation and its graceful exit.

The complete resolution of a Big Bang singularity now lies in a rather milder question; how the boost orbifold instability can be tamed in future. Our setup provides a simple stringy framework where these questions can be discussed towards understanding the origin of our universe.

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